

Article

Locational pricing to mitigate voltage problems caused by high PV penetration

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Version May 11, 2015 submitted to *Energies*. Typeset by L^AT_EX using class file mdpi.cls

Abstract: In this paper, a locational marginal pricing algorithm is proposed to control the voltage in unbalanced distribution grids. The increasing amount of photovoltaic (PV) generation installed in the grid may cause the voltage to rise to unacceptable levels during periods of low consumption. With locational prices, the distribution system operator can steer the reactive power consumption and active power curtailment of PV panels to guarantee a safe network operation. Flexible loads also respond to these prices. A distributed gradient algorithm automatically defines the locational prices that avoid voltage problems. Using these locational prices results in a minimum cost for the distribution operator to control the voltage. Locational prices can differ between the three phases in unbalanced grids. This is caused by a higher consumption or production in one of the phases compared to the other phases, and provides the opportunity for arbitrage, where power is transferred from a phase with a low price to a phase with a high price. The effect of arbitrage is analysed. The proposed algorithm is applied to an existing three-phase four-wire radial grid. Several simulations with realistic data are performed.

Keywords: Active power curtailment, distributed optimization, distribution system, locational marginal pricing, reactive voltage control, real-time pricing

18 1. Introduction

19 The electricity grid is going through a transition period. A high penetration of PV panels and the
20 ongoing electrification of the transport system requires new strategies for the operation and management
21 of the electricity grid. Typically the high power injection of PV panels does not coincide with periods of
22 high demand. The resulting high reverse power flow can cause a significant rise in the grid voltage.
23 The maximum amount of PV generation that can be connected to a Low Voltage (LV) network is
24 typically limited by this voltage rise [1,2]. In current regulations, a PV panel has to disconnect from
25 the distribution grid as soon as the maximum voltage is reached. However this can lead to unnecessary
26 curtailed green energy to keep the voltage within limits. Traditionally, Distribution System Operators
27 (DSOs) are responsible for keeping the grid voltage within limits, and today more advanced methods
28 may be needed to control the grid voltage.

29 Different control strategies have been proposed to control the grid voltage and avoid damage to the
30 grid. One method consists of PV panels that curtail part of the active power to reduce the voltage [1,2].
31 Another option is to use the remaining inverter capacity of a PV panel to do reactive voltage control
32 [3–6]. Furthermore, flexible loads like electric vehicles can increase or decrease their consumption to
33 regulate the grid voltage [7]. All of these methods are effective in managing the grid voltage, but do
34 not give a real-time incentive to the customers to control the voltage. Also these strategies should be
35 combined to achieve optimal grid voltage control and the most cost effective option should be chosen to
36 comply with the voltage limits.

37 Real-time pricing is a well known demand side management technique. When real-time pricing is
38 applied, electricity consumers are charged with prices that can vary over short time intervals. It can
39 be very effective in shaping the customers' demand [8] and can be used to keep the total consumption
40 level below the power generation capacity [9]. It is an incentive that is offered by the grid operator
41 and is assumed to be accepted by the users [10,11]. In this work a real-time pricing strategy is used to
42 control the grid voltage. A community of cooperative consumers is assumed. In contrast to the methods
43 described in [8,9,13], the tariff will not depend on the power but on the grid voltage. The distribution
44 system operator can adapt the real-time energy price to keep the voltage within limits. This price will
45 give an incentive to inject or consume reactive power to control the voltage, or if necessary to curtail
46 active power or adapt the consumption of the flexible load. The most cost effective solution will be
47 obtained. The prices are defined by a distributed gradient algorithm, based on a two way communication
48 system. The pricing is applied to unbalanced distribution networks, which requires special care due to
49 the neutral point shifting. In previous work this pricing strategy was tested for active power only [12].
50 In this work incentives will be given for reactive voltage control as well.

51 Centralizing all of the information to obtain the optimal setpoints or the real-time prices should be
52 omitted to protect the privacy of the customers [9,13–16]. Several distributed algorithms are proposed
53 to schedule loads without centralizing all off the information. In [9,13,14] the distributed algorithm is
54 based on Lagrange relaxation. [15] describes distributed algorithms that use Q-learning and Lyapunov
55 optimization. In [16] a distributed algorithm based on a non-cooperative Stackelberg game is presented.
56 In our work, network prices are defined in a distributed way by means of Lagrange relaxation.

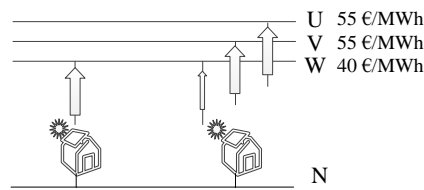


Figure 1. Representation of the arbitrage by a balancing PV inverter. When the price is higher in phase U and V than in phase W, more power will be injected into these phases. The width of the arrows represents the amount of active power flowing through the connection.

Finally arbitrage will be analysed. Arbitrage is possible when the same asset, in this case energy, does not trade at the same price at different locations. PV generation is not necessarily spread equally across the three phases. This can lead to higher voltages in the phase with the highest power production [17] and therefore a lower price for energy that is injected into this phase. When there is a price difference for the energy in the three different phases, at the same location, power can be transferred from the phase with lowest price to the phase with the highest price. This can be done with adapted PV inverters. PV inverters rarely operate at their maximal power production. If a three-phase PV inverter consists of three single-phase inverters with a common DC-bus, it is possible for the majority of the produced power to be injected into the phase with the highest power consumption, or to transfer power from highly loaded to less loaded phases, without overloading the PV inverter. This principle is illustrated in Fig. 1.

This paper is structured as follows: in Section 2 the distribution grid used in the simulation results is described and special attention is given to effects in unbalanced grids, because these will have implications on the locational grid prices. Section 3 describes the system model that defines the optimal response of the flexible loads and PV units and Section 4 elaborates on the distributed pricing strategy that results in the same response as the optimization problem which was defined in Section 3. Finally Section 5 presents some results and shows how PV panels, flexible loads and three-phase PV inverters that perform arbitrage react to the locational prices.

2. Simulated network

An existing three-phase four-wire radial distribution system with a TT earthing arrangement in Belgium was used for the simulations. The network consists of 62 customers and is depicted in Fig. 2. This network is a semi-urban reference network used in the LINEAR project [18] and has been studied often [4,7,19,20].

The main feeder cables are of type EAXVB 1 kV $4 \times 150 \text{ mm}^2$ while the cable between node A and node B is of type EAXVB 1 kV $4 \times 95 \text{ mm}^2$. The assumed operating temperature is 45°C . All households have a single-phase connection, except households 41 and 62 which have a three-phase connection to the network. The households with a single-phase connection are spread equally across the three phases in the order U,V,W,U,V,W, etc... The voltage at the secondary side of the transformer is considered to be 230 V during no load. All households have a PV installation. The PV inverter rating of the households connected to phase U and V equals 2.2 kW, while the households connected to phase W have a rating

of 3.3 kW. These assumptions will create unbalance in the network. The households with a three-phase connection have a three-phase PV installation with a rating of 6 kW.

A remarkable and important effect in three-phase four-wire grids is the neutral point shifting [21]. When a single-phase load consumes active or reactive power, a current will flow through the neutral conductor. This results in a voltage drop over the impedance of the neutral conductor and the neutral voltage experienced by all customers will shift. As a consequence of the neutral shift, reactive power absorption in phase U significantly increases the phase voltage of phase W and decreases the phase voltage of phase V. To decrease the voltage in one phase it can be more beneficial to inject reactive power into another phase than to absorb reactive power into this specific phase itself [4]. This is important for the locational pricing approach that will be developed. When voltage problems occur in one phase, the DSO should give an incentive to inject reactive power into another phase. Another consequence of the neutral shift is that consuming power in one phase, will decrease the voltage in this phase, whereas the voltage in the other two phases will slightly increase.

Voltage limits are the major concern when integrating distributed generation in distribution networks. Like in DC power flow models [17], AC models can be approximated with a linear model to describe the influence of PV panels and flexible loads on the voltage magnitude [19,22–26]. The voltage at a node m can be approximated by:

$$|V_m| \approx |V_m^{\text{base}}| + \sum_{k=1}^N \left(\mu_{m,k,i}^P P_{k,i} + \mu_{m,k,i}^Q Q_{k,i} \right) \quad (1)$$

where

- $\mu_{m,k,i}^P$ is the sensitivity of the voltage magnitude in node m by active power injected at node k into phase i ;
- $P_{k,i}$ is the active power injected or consumed at node k into/from phase i by a PV panel or a flexible load;
- $\mu_{m,k,i}^Q$ is the sensitivity of the voltage magnitude in node m by reactive power injected at node k into phase i ;
- $Q_{k,i}$ is the reactive power injected at node k into phase i by a PV panel;
- V_m^{base} is the voltage at node m due to the uncontrollable load of the households;

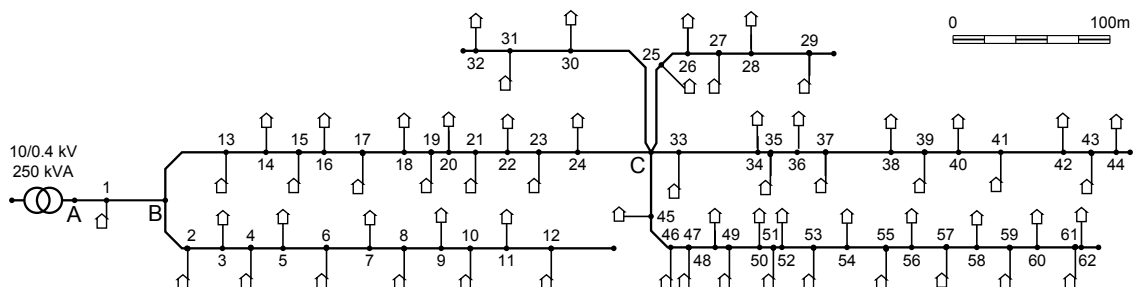


Figure 2. The network used in the simulations. All lengths are drawn to scale.

- V_m is the expected voltage at node m ;
- N is the number of nodes;

The voltages are limited between a minimum and a maximum voltage:

$$V^{\min} \leq |V_m| \leq V^{\max} \quad (2)$$

where V^{\min} and V^{\max} are the minimum and maximum allowed voltages. In this work V^{\min} and V^{\max} are chosen to be $\pm 10\%$ of the nominal voltage of 230 V. The voltages of control points at the end of the feeders have to be monitored and controlled, as these are subject to the largest voltage deviations. These voltages are measured and communicated to the DSO. In this work nodes 44 and 62 are these controlled nodes.

Using the linear voltage model described in (1) has various advantages. First of all, the DSO does not need to know the actual uncontrollable load of the households to approximate the voltage caused by the uncontrollable load alone. Since this could contain privacy sensitive information the customers preferably do not share this information with a central instance [10]. If the DSO knows the consumption of the PV panels and of the flexible loads during the voltage measurement, he can calculate the effect these had on the voltage measurement with the voltage sensitivity factors. With this information he can then obtain the voltage caused by only the uncontrollable base load V_m^{base} , without having information on the uncontrollable household consumption.

The second advantage is that the voltage constraints (1) and (2) remain an easy to handle convex set. Another advantage is that these sensitivity factors can be approximated based on historic smart meter data, without having information about the exact grid topology [24].

In real-life conditions the uncontrollable load can vary. The obtained V_m^{base} is therefore an estimate of the voltage, caused by only the uncontrollable base load, at the next time step. Furthermore, in the linear model, linearization errors should also be taken into account. It is therefore advised to include a small extra conservative margin in the limits of (2). V^{\min} and V^{\max} can be chosen to be $\pm 9\%$ of the nominal voltage, whereas the actual limits equal $\pm 10\%$ of the nominal voltage. The magnitude of the linearization errors will depend on the grid topology and the applied load profiles. Typical voltage standards, like EN50160 [27], limit the voltage deviations of the 10-minute mean RMS voltage to $\pm 10\%$. Therefore, even if the linearization errors would exceed 1%, a regular network price update can compensate quickly for these linearization errors to keep the 10-minute mean RMS voltage within limits, even with inaccurate sensitivity factors [26].

3. System model

The purpose of the DSO is to keep the voltage within limits in an optimal way, without hindering the normal market operation. To do this, it will have to steer the consumption of flexible loads and single- and three-phase PV panels.

The flexible consumption is modelled by utility functions. The utility function reflects the customer satisfaction for the consumption of their flexible loads. The higher their satisfaction, the higher the price

they are willing to pay for the requested energy. In this work quadratic utility functions are considered, which are one of the mostly used utility functions [9,28–30]:

$$U(P_{k,i}^{\text{flex}}) = \omega_k P_{k,i}^{\text{flex}} - \frac{\beta_k}{2} (P_{k,i}^{\text{flex}})^2 \quad \text{for } 0 \leq P_{k,i}^{\text{flex}} \leq P_k^{\text{max}} \quad (3)$$

140 Where

- 141 • $P_{k,i}^{\text{flex}}$ is the flexible power consumption of the load connected to node k at phase i ;
- 142 • β_k and ω_k are confidential parameters characterizing customer types;
- 143 • P_k^{max} is the maximal consumption of the flexible load connected to node k ;

For an announced price Λ_k , each customer determines the optimal $P_{k,i}^{\text{flex}}$ from:

$$\min_{P_{k,i}^{\text{flex}}} -U(P_{k,i}^{\text{flex}}) + \Lambda_k P_{k,i}^{\text{flex}} \quad (4)$$

144 It can be proven that a quadratic utility function leads to a consumption $P_{k,i}^{\text{flex}}$ that is linearly dependent
 145 on the electricity price [31]. Typically the available flexibility depends on the time of the day. During
 146 the day, consumers are often absent and the available flexibility is small. In the evening, the amount
 147 of flexibility is higher. Therefore the confidential parameters ω_k and β_k are chosen such that a price of
 148 0 €/MWh results in a consumption of 1 kW and a price of 100 €/MWh results in a consumption of 0
 149 kW between 08:00 and 18:00 for all houses with a house number which is a multiple of five. The other
 150 houses are assumed to have no available flexibility at that moment. In the evening, the parameters are
 151 chosen as such that a price of 0 €/MWh results in a consumption of 3 kW and a price of 100 €/MWh
 152 results in a consumption of 0 kW for all houses with a number which is a multiple of 2. Fig. 3 gives a
 153 summary of the responsiveness of the loads at different moments. The price Λ_k is the electricity price
 154 charged by the provider. This price consists of the generation cost, the taxes and the fixed network tariffs.
 155 Further on in this work a variable network price will be added to control the grid voltage. The price Λ_k
 156 can differ between providers. For the ease of simplicity all of the households received the same price
 157 Λ_k in this work. However, results can be generalized to a situation where all customers have a different
 158 electricity price Λ_k .

159 Single-phase PV units will always inject the produced power into their phase of connection. Part of
 160 the produced power can be curtailed to support the network, but the PV unit will never consume power.
 161 The remaining capacity of the PV inverter can be used to inject or absorb reactive power.

162 Three-phase PV installations inject power into each of the three phases. The three-phase PV inverter
 163 usually injects the same amount of active and reactive power into each phase. However when the
 164 three-phase inverter consists of three single-phase units with a common DC-bus, the unit can inject a
 165 different amount of active or reactive power into each phase. This type of inverter is referred to as a
 166 balancing inverter.

167 The DSO will try to optimally steer the flexible loads and PV units to avoid voltage limit violations
 168 in the grid. If all of the information of the flexible loads and the PV panels could be centralized at one
 169 location, the DSO would solve the following problem:

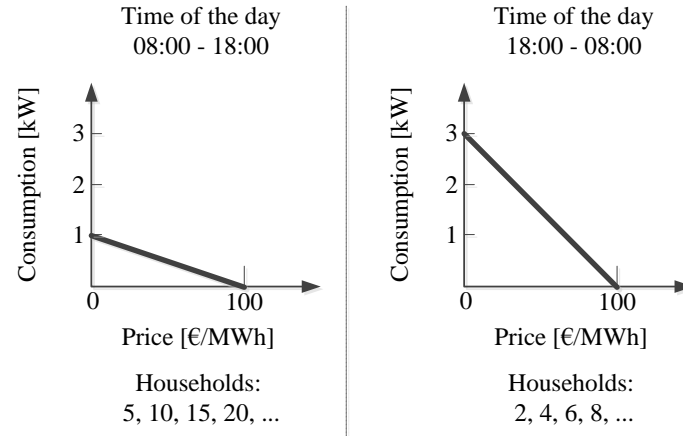


Figure 3. Response of the flexible loads on the electricity price for different moments of the day

$$\min_{P_{k,i}^{\text{flex}}, P_{k,i}^{\text{PV,curt}}, Q_{k,i}^{\text{PV}}} \underbrace{\sum_{k \in \{\theta_{\text{flex}}\}} -U(P_{k,i}^{\text{flex}}) + \Lambda_k P_{k,i}^{\text{flex}}}_{\text{Flexible loads}} + \quad (5)$$

$$\underbrace{\sum_{k \in \{\theta_1\}} -\Lambda_k (P_{k,i}^{\text{PV}} - P_{k,i}^{\text{PV,curt}}) + \alpha (Q_{k,i}^{\text{PV}})^2}_{\text{Single-phase PV}} + \quad (6)$$

$$\underbrace{\sum_{k \in \{\theta_3\}} \sum_{i \in \{U,V,W\}} \left(-\Lambda_k (P_{k,i}^{\text{PV}} - P_{k,i}^{\text{PV,curt}}) + \alpha (Q_{k,i}^{\text{PV}})^2 \right)}_{\text{Three-phase PV}} \quad (7)$$

subj. to

$$P_{k,i}^{\text{PV}} - P_{k,i}^{\text{PV,curt}} \geq 0 \quad k \in \theta_1 \quad (8)$$

$$\sum_{i \in \{U,V,W\}} (P_{k,i}^{\text{PV}} - P_{k,i}^{\text{PV,curt}}) \geq 0 \quad k \in \theta_3 \quad (9)$$

$$(Q_{k,i}^{\text{PV}})^2 + (P_{k,i}^{\text{PV}} - P_{k,i}^{\text{PV,curt}})^2 \leq S_{k,i}^2 \quad k \in \theta_1 \quad (10)$$

$$(Q_{k,i}^{\text{PV}})^2 + (P_{k,i}^{\text{PV}} - P_{k,i}^{\text{PV,curt}})^2 \leq S_{k,i}^2 \quad k \in \theta_3 \quad (11)$$

$$V^{\min} \leq |V_m| \leq V^{\max} \quad m \in N_{\text{contr}} \quad (12)$$

$$|V_m| = |V_m^{\text{base}}| + \sum_{k=1}^N (\mu_{m,k,i}^P (-P_{k,i}^{\text{PV}} + P_{k,i}^{\text{PV,curt}} + P_{k,i}^{\text{flex}})) + \sum_{k=1}^N \mu_{m,k,i}^Q Q_{k,i}^{\text{PV}} \quad (13)$$

170 Where

- 171 • θ_{flex} is the set of all customers with a flexible load unit;
- 172 • θ_1 is the set of all customers with a single-phase PV unit;
- 173 • θ_3 is the set of all customers with a three-phase PV unit;

- $P_{k,i}^{\text{PV}} - P_{k,i}^{\text{PV,curt}}$ is the net injected power into phase i of the PV panel connected at node k . For a single-phase PV unit $P_{k,i}^{\text{PV}}$ is the total produced power, for a three-phase unit $P_{k,i}^{\text{PV}}$ is one third of the total produced power. Part of the produced power can be curtailed $P_{k,i}^{\text{PV,curt}}$;
- $Q_{k,i}^{\text{PV}}$ is the reactive power injected/absorbed by the PV panel connected at node k into phase i ;
- $S_{k,i}$ is the inverter rating of the PV unit connected at node k to phase i . For a three-phase unit $S_{k,i}$ is one third of the total three-phase inverter rating;
- α is a parameter to penalize reactive power injection or absorption by a PV panel;

The objective function consists of three terms. The first term (5) maximizes the utility of the flexible loads. The second term (6) reflects the income of the single-phase PV units. The units get a price of Λ_k for the injected power and have a decreased income when they have to curtail power. They can also provide reactive power, but this at a small cost characterised by the parameter α . This cost should account for increased losses due to reactive voltage control. α is chosen to be $1 \text{ €/Mvar}^2\text{h}$. The last term (7) of the objective function gives the income of the three-phase PV units. It consists of the income for injecting active power and a penalty for reactive voltage control.

A small penalty term is added to the objective function that penalizes the balancing inverter for injecting a different amount of active power into the three phases. This term ensures that when there are no voltage problems the same amount of power is injected in each phase. This term is small compared to the other terms and for the ease of simplicity this term is not presented in the objective function.

Constraint (8) ensures that a single-phase PV unit does not curtail more energy than the produced amount. Constraint (9) ensures the same for a three-phase PV unit. The amount of reactive power absorbed or injected by a single-phase PV unit is limited by constraint (10), while constraint (11) limits the reactive power by a three-phase unit. When the three-phase inverter does not consist of three single-phase units, the active and reactive power injection in each phase have to be equal. This can be implemented by adding the following constraints:

$$\begin{aligned} P_{k,U}^{\text{PV}} &= P_{k,V}^{\text{PV}} = P_{k,W}^{\text{PV}} \\ Q_{k,U}^{\text{PV}} &= Q_{k,V}^{\text{PV}} = Q_{k,W}^{\text{PV}} \end{aligned} \quad (14)$$

Constraint (12) guarantees that the voltage will stay within limits for all the control nodes N_{contr} .

The solution of this problem will optimally control the flexible loads and PV units. If no voltage problems occur, the PV panels will not curtail any energy or provide reactive power. Also the flexible loads will behave in their normal way as described by (4). If voltage problems occur, this will change.

Centralizing all information at one location to solve the DSO optimization problem might be complicated. Besides that, privacy sensitive information like the utility function is preferably not shared with a central instance. Therefore there is a need to create a distributed pricing algorithm, that by means of network prices results in the same optimal solution, but that does not require all the information to be gathered at one place. This distributed algorithm will rely on duality theory. The DSO optimization problem can be reformulated as a decomposable dual problem and can be solved using a dual ascent method, with the same solution. Strong duality holds because the primal problem is convex and a strictly feasible point will exist. Dual ascent methods rely on an iterative update of the Lagrange multiplier to

obtain the same solution. These methods are also called Lagrange Dual Decomposition methods and are commonly applied in power systems [9,12,33]. Other distributed algorithms have been proposed to control the reactive power contribution of PV inverters [34], but these do not make use of a real-time pricing scheme.

The voltage in the network is controlled by constraint (12). The Lagrange multipliers Λ^{DSO} of constraint (12) have an economical interpretation. They equal the shadow price for creating voltage problems in the control node. This is a price per Volt. To find the price per unit of active or reactive power, one has to multiply this price per Volt by the influence of active or reactive power on the voltage magnitude:

$$\underbrace{\Lambda_m^{DSO}}_{\text{Price per Volt}} \xrightarrow[\text{active power}]{\text{Influence on the voltage of}} \underbrace{\Lambda_m^{DSO} \mu_{m,k,i}^P}_{\text{Price per kWh}} \quad (15)$$

$$\xrightarrow[\text{reactive power}]{\text{Influence on the voltage of}} \underbrace{\Lambda_m^{DSO} \mu_{m,k,i}^Q}_{\text{Price per kVarh}} \quad (16)$$

The parameters $\mu_{m,k,i}^P$ and $\mu_{m,k,i}^Q$ express the influence that a node k has on the voltage of the control node m. They differ between different locations and therefore they can differ between different customers. Charging these shadow prices $\Lambda_m^{DSO} \mu_{m,k,i}^P$ and $\Lambda_m^{DSO} \mu_{m,k,i}^Q$ will result in optimal system behaviour. In the next section will be discussed how these shadow prices can be found without centralizing all information. The dual ascent method applied for this will consist of an iterative update of the Lagrange multipliers, which in this case coincide with the network prices.

4. Distributed pricing algorithm

An iterative distributed algorithm will solve the dual of the DSO optimization problem by iteratively updating the Lagrange multipliers of the voltage constraints. The Lagrange multipliers are the shadow prices for creating voltage problems in the control nodes. These should be charged to the customers to obtain the optimal solution. This price is found by an iterative scheme. Every iteration, the flexible loads and PV units receive a network price from the DSO. They respond back to the DSO how they would react on this network price. Based on this information the DSO can update the network price and send this updated price back to all the PV units and flexible loads. This until the price has converged.

A flexible load will define its planned consumption based on the following problem:

$$\min_{P_{k,i}^{\text{flex}}} -U(P_{k,i}^{\text{flex}}) + \Lambda_k P_{k,i}^{\text{flex}} + \quad (17)$$

$$+ \sum_{m=1}^{N_{\text{contr}}} \Lambda_m^{DSO} \mu_{m,k,i}^P P_{k,i}^{\text{flex}} \quad (18)$$

Compared to (4) an extra network price $\Lambda_m^{DSO} \mu_{m,k,i}^P$ is added. The price for making use of the network depends on the location and phase of the customer. In case voltage problems occur in a control node, a price Λ_m^{DSO} is set for using voltage "resources" in this control node, and the customer is charged depending on their influence $\mu_{m,k,i}^P$ on this control node. Due to the neutral point shift, the sign of $\mu_{m,k,i}^P$ can be both positive and negative, dependent on the phase of connection. Therefore consumption can

both be rewarded and penalized by the DSO. N_{contr} is the number of control nodes. In this work there are two control nodes: node 44 and 62. The voltages of all the three phases of these nodes are controlled.

PV installations will also respond to electricity prices. They can curtail active power or provide reactive voltage control to support the network. A single-phase PV unit will define its active and reactive power set point based on the following problem:

$$\begin{aligned}
 \min_{P_{PV,curt}, Q_{PV}} \quad & -\Lambda_k (P_{k,i}^{PV} - P_{k,i}^{PV,curt}) + \alpha (Q_{k,i}^{PV})^2 + \\
 & - \sum_{m=1}^{N_{contr}} \Lambda_m^{DSO} \mu_{m,k,i}^P (P_{k,i}^{PV} - P_{k,i}^{PV,curt}) + \\
 & + \sum_{m=1}^{N_{contr}} \Lambda_m^{DSO} \mu_{m,k,i}^Q Q_{k,i}^{PV} \\
 \text{subj. to} \quad & (Q_{k,i}^{PV})^2 + (P_{k,i}^{PV} - P_{k,i}^{PV,curt})^2 \leq S_k^2 \\
 & P_{k,i}^{PV} - P_{k,i}^{PV,curt} \geq 0
 \end{aligned} \tag{19}$$

$$\tag{20}$$

$$\tag{21}$$

An extra locational dependent network price is added compared to the normal objective function defined by (6). The same price Λ_m^{DSO} is set for using voltage "resources" in this control node, and the customer is charged depending on their influence $\mu_{m,k,i}^P$ and $\mu_{m,k,i}^Q$ on this control node. Note that $\mu_{m,k,i}^P$ is not equal to $\mu_{m,k,i}^Q$, because active power has a different influence on the voltage of the control node as reactive power. Therefore the prices for active power are not identical to the prices for reactive power. When analysing this objective function it is clear that the total price for active power is the sum of the electricity price of the provider and a variable price which depends on the shadow price of the grid voltage $\Lambda_k + \sum_{m=1}^{N_{contr}} (\Lambda_m^{DSO} \mu_{m,k,i}^P)$. As long as this total price for the energy provided is positive, no active power will be curtailed. When there is no reward for providing or absorbing reactive power, the PV units will not provide reactive voltage control.

A three-phase PV unit will define its active and reactive power set point in each phase based on the following problem:

$$\begin{aligned}
 \min_{P_{PV,curt}, Q_{PV}} \quad & \sum_{\substack{i \in \\ \{U, V, W\}}} -\Lambda_k \left((P_{k,i}^{PV} - P_{k,i}^{PV,curt}) + \alpha (Q_{k,i}^{PV})^2 \right) + \\
 & - \sum_{m=1}^{N_{contr}} \sum_{\substack{i \in \\ \{U, V, W\}}} \Lambda_m^{DSO} \mu_{m,k,i}^P (P_{k,i}^{PV} - P_{k,i}^{PV,curt}) \\
 & + \sum_{m=1}^{N_{contr}} \sum_{\substack{i \in \\ \{U, V, W\}}} \Lambda_m^{DSO} \mu_{m,k,i}^Q Q_{k,i}^{PV} \\
 \text{subj. to} \quad & (Q_{k,i}^{PV})^2 + (P_{k,i}^{PV} - P_{k,i}^{PV,curt})^2 \leq S_{k,i}^2 \quad i \in U, V, W \\
 & \sum_{i \in \{U, V, W\}} (P_{k,i}^{PV} - P_{k,i}^{PV,curt}) \geq 0
 \end{aligned}$$

Compared to the single-phase PV panels three-phase PV panels will receive a network price for each phase. Dependent on the phase of connection of node m , $\mu_{m,k,i}^P$ and $\mu_{m,k,i}^Q$ can be both positive and negative depending on their phase i . This gives an incentive to transfer power from one phase to another.

Once the flexible loads and PV panels have calculated their planned consumption for the given price, they will send this information to the DSO. They do not yet adopt this consumption as they have to wait for the DSO to inform them that the price has converged. It is assumed that the planned consumption for the given price is a binding agreement. Therefore, when the DSO informs the flexible loads and PV panels of the converged price, they will have to adopt the proposed consumption levels.

With the planned consumption of each unit, the DSO can calculate the expected voltage magnitude of the control nodes if these plans would be realised:

$$|\widehat{V}_m| \approx |V_m^{\text{base}}| + \sum_{k=1}^N (\mu_{m,k,i}^P (-P_{k,i}^{\text{PV}} + P_{k,i}^{\text{PV,curt}} + P_{k,i}^{\text{flex}})) + \sum_{k=1}^N \mu_{m,k,i}^Q Q_{k,i}^{\text{PV}} \quad (22)$$

The expected voltage should respect the voltage limits. If this voltage is outside the limits, the network price should be increased. If the voltage is clearly inside the limits, the network price might have been too high and can be reduced. As discussed earlier, the network price corresponds to the Lagrange multipliers of constraint (12). Only one of the constraints can be active: either the upper voltage limit is reached, or the lower voltage limit is reached. In case the voltage becomes too high, the update rule of the Lagrange multiplier becomes:

$$\widehat{\Lambda}_m^{DSO} = \Lambda_m^{DSO} + \gamma (\widehat{V}_m - V^{\text{max}}) \quad (23)$$

$$\Lambda_m^{DSO} = \max(\widehat{\Lambda}_m^{DSO}, 0) \quad (24)$$

If the voltage has dropped below the limits, the update rule becomes:

$$\widehat{\Lambda}_m^{DSO} = \Lambda_m^{DSO} + \gamma (V^{\text{min}} - \widehat{V}_m) \quad (25)$$

$$\Lambda_m^{DSO} = -\max(\widehat{\Lambda}_m^{DSO}, 0) \quad (26)$$

This update rule is a gradient ascent method to find the optimal Lagrange multipliers [32]. Λ_m^{DSO} will only differ from zero when network limits are reached in node m . One iteration consists of a price update from the DSO, followed by a response from all the customers. Fig. 4 presents this loop. Only once the price has converged, end-users will be informed that the price has converged and then they will adapt their consumption. In the preliminary iterations the planned consumption is communicated for the given price, but this consumption is not actually adopted. In this work a constant stepsize γ is used to update the Lagrange multipliers. Convergence with a constant stepsize is within a near-optimal ball, but it is typically faster than convergence with a diminishing stepsize [13,32]. To further improve convergence, a quadratic term is added to the single-phase PV optimization problem that penalizes the deviation from the calculated curtailed PV power in the previous iteration. This limits the oscillatory behavior from one iteration to the next [35].

The only information that the system operator exchanges with the customers is a network price (for active and reactive power), while each customer responds his planned consumption level for this price. Privacy sensitive information, like the customer utility function, is not shared with the system operator.

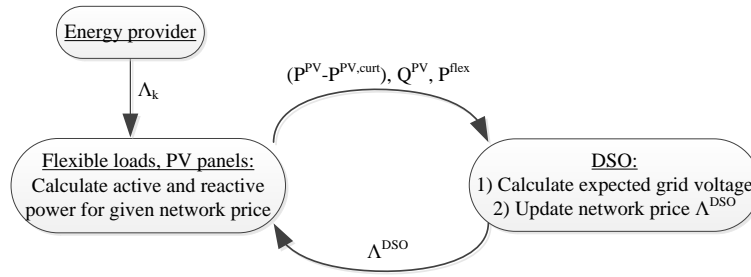


Figure 4. Flowchart of the distributed locational pricing scheme.

The system operator also needs a real-time voltage measurement of the grid voltage of the control nodes. Only real-time information is used. Future work could include predictions in the algorithm.

5. Results

5.1. Simulation of one time step of 10 minutes

The pricing algorithm is tested on the network of Fig. 2. The price for electricity Λ_k , excluding the network price is defined as 50 €/MWh. Consumers can have different providers that charge different electricity prices, but in this work all of the consumers are assumed to have an equal fixed electricity price. In the first simulation, the algorithm is evaluated for one single time step of 10 minutes. This is a time step with a (high) PV production of 90 % of the inverter rating. The load is chosen randomly between 0.5 and 0.7 kW for nodes connected to phase U and between 0 and 0.3 kW for nodes connected to phase V and W. Fig. 5 presents the voltage in the network for these conditions when no pricing algorithm is applied. The high PV production leads to an unacceptably high voltage in the nodes at the end of the feeder connected to phase W. Adding a network price will avoid this high voltage, by giving incentives to curtail energy, to provide reactive power or to transfer power from one phase to another.

5.1.1. Only active power curtailment

When the only form of voltage control is the active power curtailment by PVs and the response of flexible loads, network prices will have to be high. PV units only curtail active power when the network price exceeds the price they would normally receive for the produced energy. The resulting voltage in the network is shown in Fig 6a. The final network price $\sum_{m=1}^{N_{contr}} \Lambda_m^{DSO} \mu_{m,k,i}^P$ that mitigates the voltage problems is presented in Fig. 6b. As can be seen the network price can drop below -50 €/MWh for nodes connected to phase W at the end of the feeder, making the total price for electricity negative in these nodes. This means that consumers get rewarded for electricity consumption. The price for power consumption only drops for nodes connected to phase W. This is the phase with the highest PV production. Due to the neutral point shifting effect, power consumption in phases U and V can increase the voltage in phase W. Therefore the price for nodes connected to these phases increases. The nodes

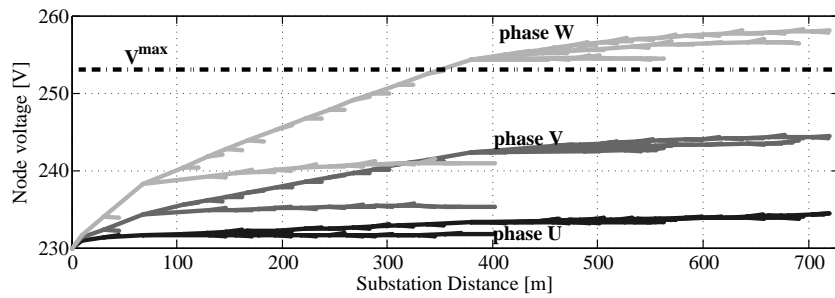


Figure 5. Grid voltages when no pricing scheme is used to control the voltage.

with the highest influence on the voltage of phase W of the control nodes receive the highest network price. These nodes will curtail active power. The response of the flexible loads and the PV units is also given in Fig. 6b. Flexible loads connected to phase W will increase their consumption, whereas the other flexible loads will decrease their consumption. Also note that the nodes with a three-phase PV unit do not curtail power. Arbitrage is not allowed and it is assumed that an equal amount of active power is injected into each phase. The penalty that needs to be paid to inject power into phase W does not outweigh the money received for injecting power into phases U and V.

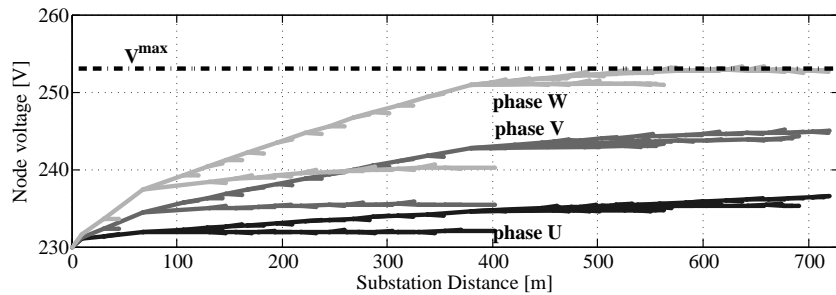
5.1.2. Active power curtailment and reactive voltage control

When reactive power can also be used to control the voltage, the extra network price will remain small. The costs associated with providing reactive power is small for PV units, because of the small α in (6) and (7). Therefore small price incentives will suffice to keep the voltage within limits. The resulting voltage in the network is shown in Fig 7a. The final price obtained by the pricing algorithm that mitigates the voltage problems is presented in Fig. 7b. The response of the flexible loads and the PV units is also given in this figure. As can be seen, no expensive active power curtailment takes place. The flexible loads have a limited adaptation of their consumption due to the relatively small price changes. The cheaper reactive power control is used to control the voltage. Three-phase PV units need to inject or absorb the same amount of reactive power into each phase

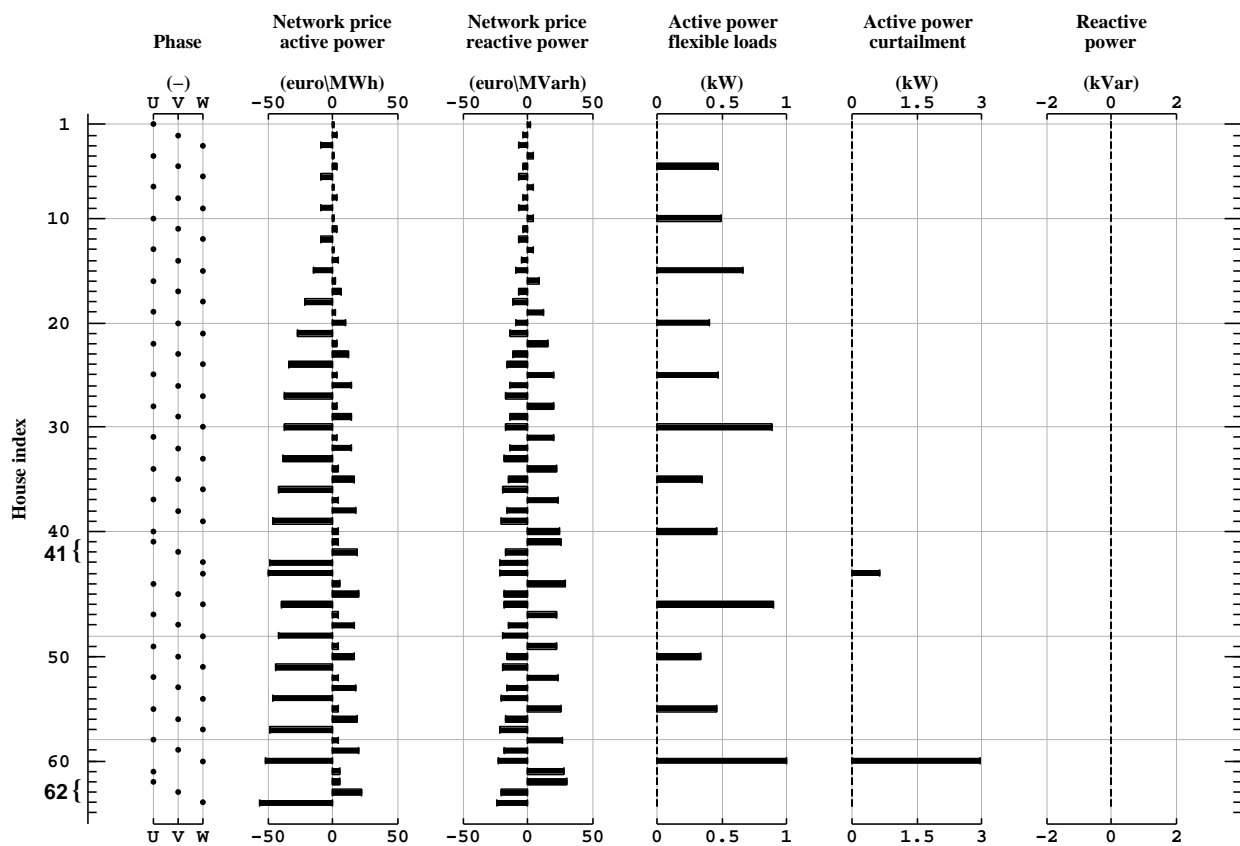
The network prices presented in Fig. 6b and 7b were obtained with the iterative scheme that was presented in Section 4. The convergence of the network price is fast. Fig. 8 shows the evolution of the network price of the three phases of node 62 for the simulation with reactive voltage control. After 35 iterations the final price is obtained. The PV production might change quickly. Therefore, during the iterations of the pricing scheme, the grid voltage might be out of limits for a short period of time. However, a short period with a voltage out of limits is not a problem, because grid standards typically only limit the 10-minute mean RMS voltage to be within $\pm 10\%$ [27]. Besides that, a small extra conservative margin in the limits of V^{max} and V^{min} will avoid that the voltage will significantly exceed the limits during a sudden change of PV generation.

5.2. Simulation of one week

In the second simulation a sunny week is analysed. 62 statistically representative residential load profiles were available to perform load flow simulations. Generation of these load profiles is described

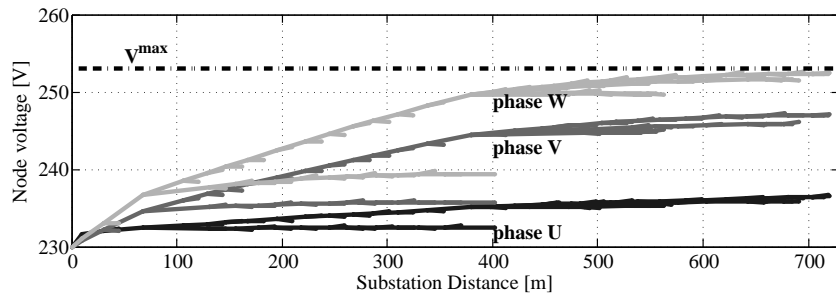


(a) Grid voltages.

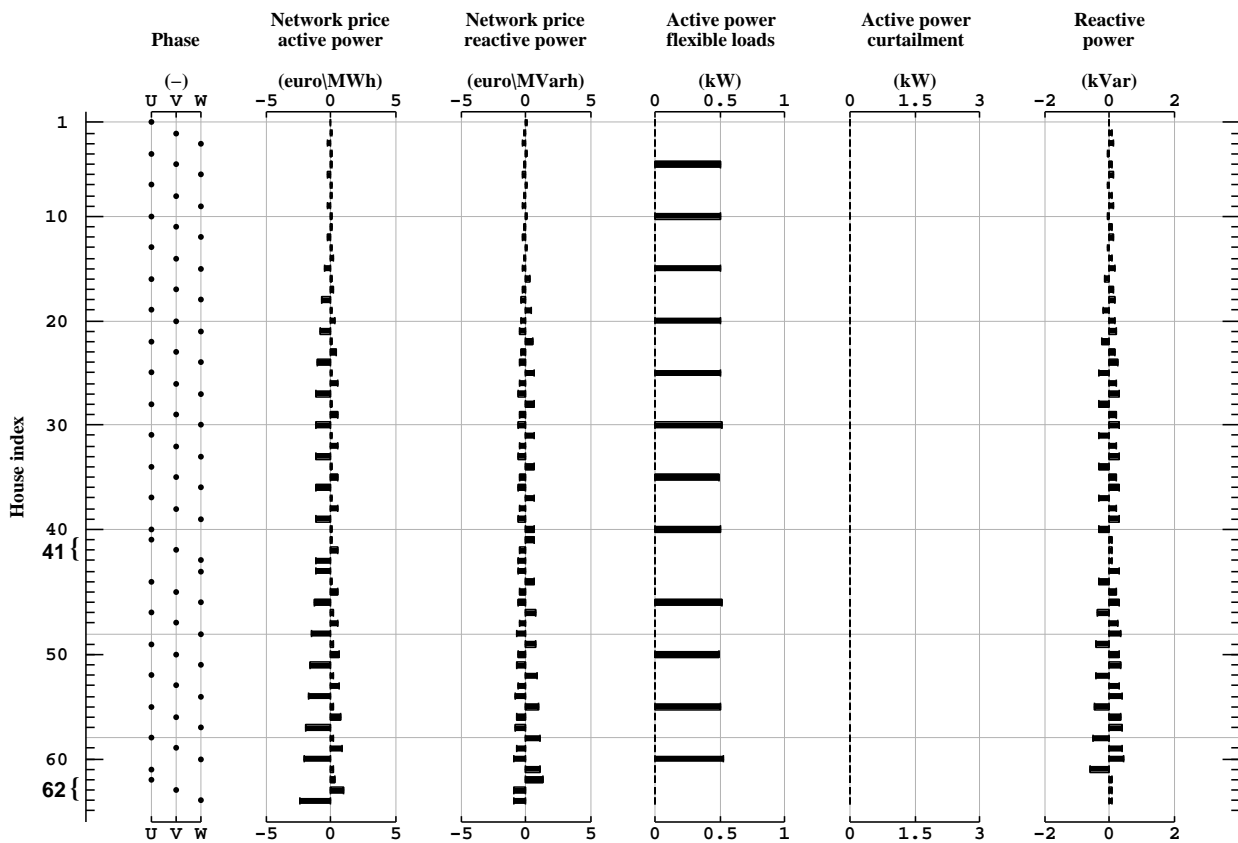


(b) Network prices, flexible consumption, curtailed PV power and reactive power provided by PV panels for each node of the grid.

Figure 6. Results of a pricing scheme that gives incentives to adapt flexible consumption and to curtail PV power.



(a) Grid voltages.



(b) Network prices, flexible consumption, curtailed PV power and reactive power provided by PV panels for each node of the grid..

Figure 7. Results of a pricing scheme that gives incentives to adapt flexible consumption or to provide reactive voltage control and to curtail PV power.

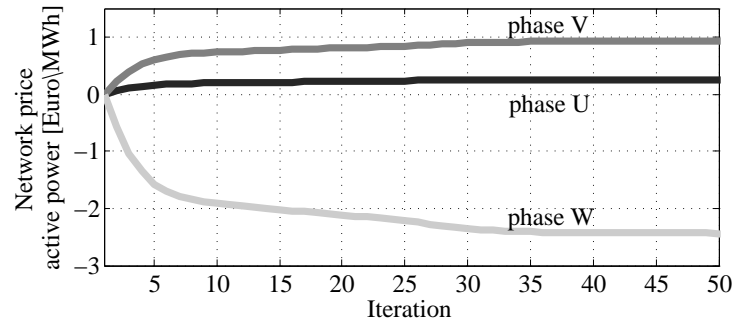


Figure 8. Convergence of the network price at node 62.

in [36]. The PV profile was measured at a fixed rooftop PV installation at KU Leuven and scaled to the inverter size. Alternatively, PV profiles could be synthetically generated [37]. The price charged by the electricity provider Λ_k was assumed to be 50 €/MWh and constant. Fig. 9a presents the minimum and maximum phase voltage in the control nodes and the extra network price charged by the DSO to mitigate voltage problems. Besides the response of the flexible loads only active power curtailment is allowed. Thanks to the pricing mechanism the voltage remains in between limits. The majority of the time the network price $\sum_{m=1}^{N_{contr}} \Lambda_m^{DSO} \mu_{m,k,i}^P$ is zero. During the day, when there is a high PV production, the price for energy consumed in phase W will drop to increase the flexible consumption and if necessary curtail active power. Sometimes a network price has to be charged to avoid an excessive drop of the grid voltage in the evening. Especially the end nodes have a very volatile price and the difference in price between the phases can become large.

When PV panels can provide reactive power the price volatility will drop significantly. Fig. 9b presents the minimum and maximum phase voltage in the control nodes and the extra network price charged by the DSO to mitigate voltage problems in this case. Again the pricing mechanism can keep the voltage in between the limits, but the network prices are reduced with a factor 10 at least. The network price has to increase/decrease until sufficient customers participate in the grid voltage control. If only active power control can be applied, the price has to increase or decrease significantly before the curtailment of PV power might become economically interesting for the owner of the installation. On the other hand, the costs associated with providing reactive power are small for PV units, because of the small α in (6) and (7). Therefore, a small price increase/decrease gives sufficient reimbursement for a lot of customers to participate in the voltage control, making the network price smaller and less volatile.

Finally we analyse the effect of arbitrage. If the three-phase PV inverters are allowed to do arbitrage, the variable network price will drop even further. This is typical for arbitrage, as it has the effect of causing prices in different locations to converge. Fig. 10a presents the minimum and maximum phase voltage in the control nodes and the extra network price charged by the DSO to mitigate voltage problems. The network price is reduced compared to Fig. 9a and Fig. 9b. Fig. 10b shows the power exchanged with each phase by the two three-phase PV inverters. During the day, a maximum amount of active power is injected into phases U and V. The remaining power is injected into the overloaded phase W. This because during the day, the network price in phase W becomes negative as can be seen in Fig. 10a. Therefore the reimbursement for injecting power into phase W is smaller. At night, power is extracted from the phases with a low load and injected into phases with a high load.

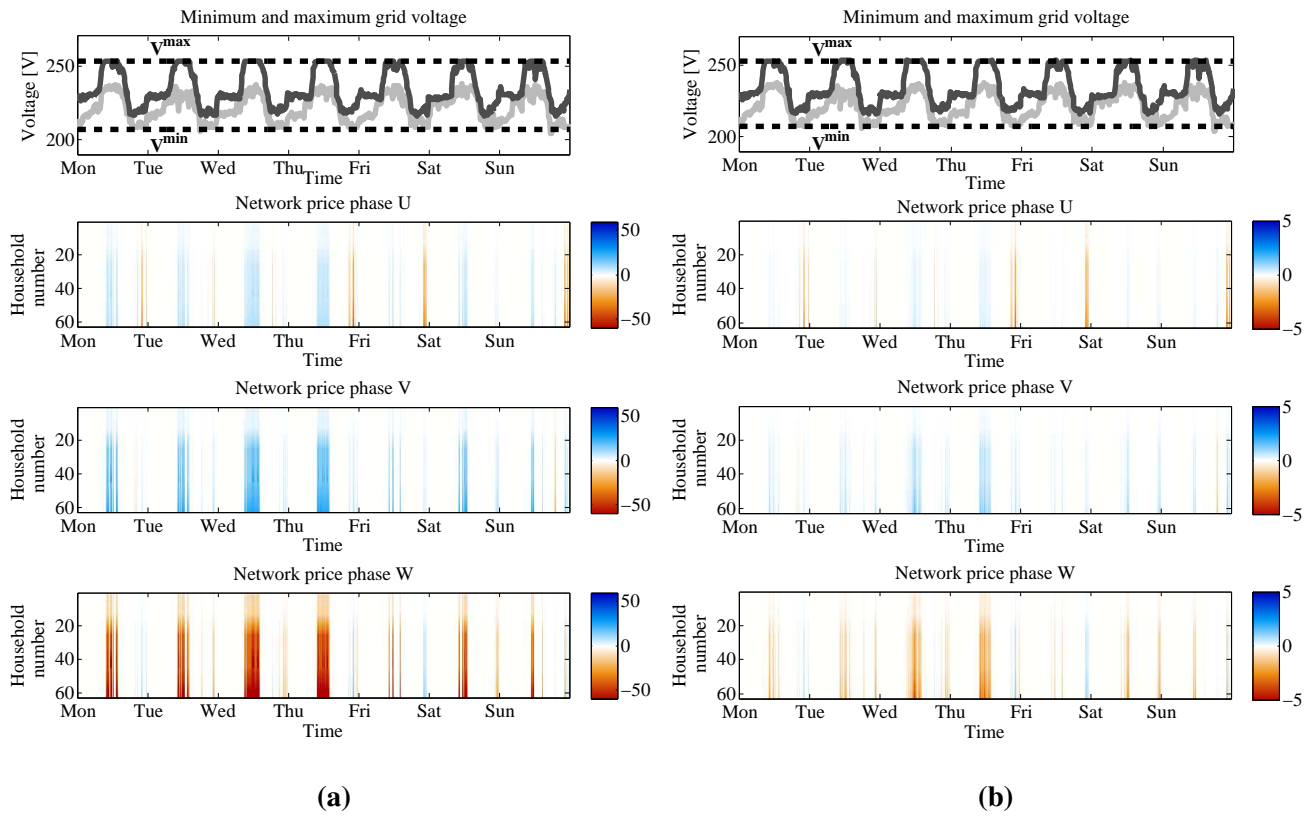


Figure 9. The maximum and minimum phase voltage that occurs in the control nodes and the network price at each node for active power during a sunny week in case of active power curtailment only **(a)** and both active power curtailment and reactive voltage control **(b)**. Note that the price scale is different.

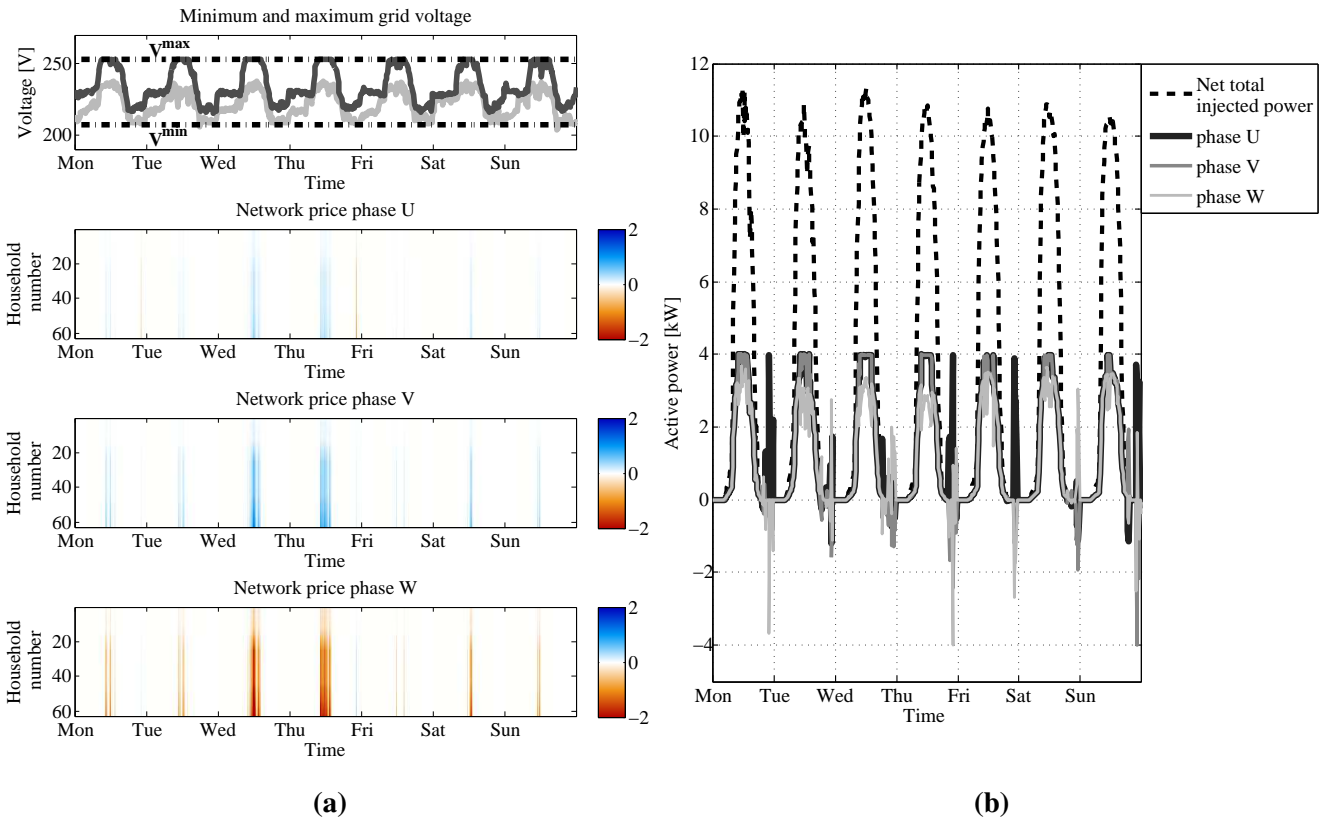


Figure 10. The maximum and minimum phase voltage that occurs in the control nodes and the network price at each node for active power curtailment during a sunny week in case of active power curtailment, reactive voltage control and arbitrage **(a)** and the total net injected power by the two balancing inverters and the total injection into each of the phases during this week **(b)**.

5.3. General remarks

In this work it was assumed that the flexible loads and PV units react to the network prices to cover their own cost for the provided network service. They could also respond strategically to the network prices to increase their profit. When only a small amount of loads and PV units react to the network prices, they can increase the network prices by agreeing to adapt their consumption only for a minimum network price. In further research the effect of this strategic behavior could be explored.

The network price can also be a control signal that is used to control the grid voltage in a distributed way. It is not necessarily charged to the customers. For example, customers participating in the voltage control can be reimbursed for the offered amount of control, for the occasions that the control is activated or by a yearly fixed fee if they participate. One of the main critics in these types of algorithms is the assumption that consumers are very sophisticated, to the level of being daily energy traders that are aware of their costs. This is absolutely not necessary in the proposed framework. A PV panel can be equipped with a control box, with standard settings for the costs.

6. Conclusions

In this paper a locational pricing algorithm that takes into account the voltage limitations of unbalanced three-phase, four-wire radial networks is proposed. When the limits of the maximum or minimum voltage are reached, the system operator defines an extra grid price to give an incentive for reactive voltage control or to curtail active power. The total price of electricity consists of a price from the provider and a grid price for using the network. The grid prices are defined by a distributed optimization problem. Due to the unbalanced nature of the network, prices can differ between the different phases at one connection point. This gives an incentive to balance the network, by transferring power from a phase with a low price to a phase with a high price.

Simulations show that the voltage can be controlled with the pricing scheme. When voltage problems occur, reactive voltage control will mainly be applied. The costs associated with reactive voltage control are smaller than those associated with active power curtailment. The price for making use of the network remains small, as small rewards give sufficient incentive for reactive voltage control. If arbitrage by three-phase PV units is added the price differences between the phases diminish. When reactive voltage control is not applied, the network price needs to rise significantly to justify the curtailment of active power.

Acknowledgments

Sam Weckx has a PhD fellowship of the Research Foundation Flanders (FWO-VITO).

Conflicts of Interest

The authors declare no conflict of interest.

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